Eclipsing Binaries in the OGLE Variable Star Catalog. II.
Light Curves of the W UMa-type Systems in Baade's Window

Slavek M. Rucinski[1]

e-mail: *rucinski@astro.utoronto.ca*

81 Longbow Square, Scarborough, Ontario M1W 2W6, Canada

February 5, 2008

ABSTRACT

Light curves of the contact systems visible in the direction of Baade's Window have been analyzed using the first coefficients of the Fourier representation. The results confirm that the geometric contact between components is usually weak. Systems showing significant differences in the depths of eclipses are very rare in the volume-limited sample to 3 kpc: only 2 among 98 contact systems show the difference larger than 0.065 mag; for most systems the difference is < 0.04 mag. If this relative frequency of 1/50 is representative, then one among 12,500 – 15,000 Main Sequence F–K spectral-type stars is either a semi-detached or poor-thermal-contact system. Below the orbital period of 0.37 day, no systems with appreciable differences in the eclipse depths have been discovered. Since large depth differences are expected to be associated with the "broken-contact" phase of the Thermal Relaxation Oscillations, this phase must be very short for orbital periods above 0.37 day and possibly entirely absent for shorter periods. In the full sample, which is dominated by intrinsically bright, distant, long-period systems, larger eclipse-depth differences are more common with about 9% of binaries showing this effect. Sizes of these differences correlate with the sense of light-curve asymmetries (differing heights of maxima) for systems with orbital periods longer than 0.4 day suggesting an admixture of semi-detached systems with accretion hot spots on cooler components. The light-curve amplitudes in the full sample as well as in its volume-limited sub-sample are surprisingly small and strongly suggest a mass-ratio distribution steeply rising toward more dissimilar components. Since the sky-field sample is dominated by contact binaries with large amplitudes, it is suggested that a large fraction of low mass-ratio systems remains to be discovered among bright stars in the sky. For a mass-ratio distribution emphasizing low values, an approach based on the statistics of the inner contact angles of totally-eclipsing systems may offer a better means of determining this distribution than the statistics of the variability amplitudes.

Subject headings: stars: close binaries - stars: eclipsing binaries – stars: variable stars

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[1]Affiliated with the Department of Astronomy, University of Toronto and Department of Physics and Astronomy, York University

1. INTRODUCTION

The first paper of this series (Rucinski 1997 = R97) demonstrated the potential of W UMa-type systems as tracers of the Galactic structure and population content. This paper should be consulted for several details on the OGLE eclipsing-binary sample and on its analysis. R97 used almost exclusively the time-independent data, such as the orbital periods, maximum brightness magnitudes and colors, and upper limits to the interstellar reddening and extinction. The present paper will concentrate on properties of contact binaries which are accessible through analysis of the light curves.

The only information from light curves used in R97 was in the preparation of an automatically selected sample of contact systems observed with relatively small errors and fulfilling shape criteria for contact configurations. The algorithm, based on Fourier decomposition of light curves for all 933 eclipsing systems in nine OGLE fields in Baade's Window (BW), defined a much smaller sample of 388 contact binaries (the "restricted-" or R-sample) than the original visual classification of the OGLE project which contained 604 systems (the O-sample). For both samples, the additional constraint was that the period should be shorter than one day and that the $V - I$ colors should be available. The current paper will present results mostly for the R-sample.

For consistency with R97, the same data from the OGLE Catalog, Parts I – III are used in this paper (Udalski et al. 1994, 1995a, 1995b). The new data for BW9 – BW11 (Udalski et al. 1996) have not been included here also because the extinction/reddening data are not available for these fields.

It has been shown in R97 that the distances of the W UMa systems in the BW are distributed rather evenly in space all the way to the Bulge. Determinations of distances required assumption of the length scale of the interstellar absorption layer along the line of sight, d_0. Two extreme assumptions on d_0 were considered: (1) extinction extending uniformly to the Bulge, $d_0 = 8$ kpc, and (2) extinction truncated at $d_0 = 2$ kpc, with the latter assumption giving a more consistent picture. The choice is however not critical for the results in the present paper and enters only through a definition of the local sample, judged to be complete to distance of 3 kpc. When applied to the R-sample, this choice will be designated R_2, in consistence with R97.

The properties of the contact binaries which were extensively discussed in R97 and will not be repeated here, although of relevance for the present paper, were the following: Most of the W UMa-type systems belong to the population of old Turn-Off-Point stars. Their periods and colors are confined to relatively narrow ranges of $0.25 < P < 0.7$ days and $0.4 < (V - I)_0 < 1.4$. The *apparent* density of contact binaries is about $(7 - 10) \times 10^{-5}$ systems per pc^3 and the *apparent* frequency, relative to nearby dwarfs of similar colors is one contact system per about 250 – 300 Main Sequence stars. A correction for undetected systems with low orbital inclination would increase the above numbers by a factor of about 2 times. Judging from these results, which were derived from the volume-limited sample, most of the contact binaries belong to the Old Disk population, but with a possibility of an admixture of Halo and Thick Disk systems.

The present paper is organized as follows: Sections 2 – 4 contain discussions of various properties of the contact binaries which can be analyzed using Fourier coefficients of the light-curve decomposition. Section 2 discusses the degree-of-contact (sometimes also called "over-contact", although such an expression seems to be awkward). Section 3 discusses the statistics of occurrence of the poor thermal contact in contact binaries. Section 4 addresses the poorly explored matter of light-curve asymmetries. Sections 5 and 6 address the matter of the mass-ratio distribution determinations from the amplitude distribution and from the distribution of angles of totality for totally eclipsing systems. Section 7 gives a summary of the paper.

2. DEGREE OF CONTACT

As was discussed in R97, the two even cosine terms of the Fourier decomposition of light curves, a_2 and a_4, can be used to separate contact binaries from detached binaries (see Figure 4 in R97). This separation was used in the definition of the R-sample, which was selected on the basis of (1) a good overall fit of the cosine representations to the light curves, with the mean standard error of the fit better than 0.04 mag (i.e. about 2-sigma error for most systems), and (2) a shape criterion based on the a_2 and a_4 coefficients. This criterion can be directly linked to to the degree-of-contact parameter used in Rucinski (1993 = R93). As was shown in R93, a crude estimate of the degree of contact can be made by interpolation in the $a_4 - a_2$ plane. Figure 1 shows the O-sample and R-sample data, compared with the theoretical predictions of R93. The three domains marked in the figure are defined as envelopes for all combinations of mass-ratios ($0.05 \leq q \leq 1$) and inclinations ($30 \leq i \leq 90$ degrees) considered in R93, for three cases of the degree of contact: when the stars just fill the inner critical envelope ($f = 0$, the steepest rising and the narrowest domain), when the stars fill the outermost common envelope ($f = 1$, the largest and lowest domain in the figure) and for one intermediate case ($f = 0.5$). The upper edge of the marginal contact domain is basically the same as the cut-off line for the Fourier filter: $a_4 = a_2 (0.125 - a_2)$. The degree of contact f is defined in terms of the potential, as in R93.

In addition to the R-sample, the O-sample is included in Figure 1 to show how many systems have been lost in moving from the original OGLE classification to the restrictive R-sample. A large fraction of rejected systems had poorly defined light curves, but some very interesting objects, very close to contact, did not pass the Fourier filter. One example of particular importance is the shortest-known period Main Sequence binary #3.038 (P = 0.198 day) which consists of two very similar, strongly distorted, but detached, M-type dwarfs (a full light-curve synthesis solution by Maceroni & Rucinski is in preparation). Since it was decided to concentrate on genuine contact systems, the R-sample will be used from this point on. However, one should realize the limitations of the selection, which was based on the Fourier coefficients calculated in R93 for one effective temperature and one spectral band (V-filter). Since the contact-binary variability is dominated by geometrical changes and weakly depends on the atmospheric properties, it can be argued that these limitations should not strongly affect any *relations* between the coefficients (as opposed to

their numerical values). However, this approach cannot really replace full synthesis solutions of the light curves, especially for derivation of the values of f.

As we can see in Figure 1, most contact systems occupy a band corresponding to moderate degrees of contact, of about $0 < f < 0.5$. This is in agreement with the previous results (Lucy 1973; Rucinski 1973; Rucinski 1985, Fig.3.1.9). However, this method of estimating f can be used only in a qualitative sense. In addition, it entirely loses sensitivity for small amplitudes and nothing can be said about cases with $|a_2| < 0.1$. In their majority, these will be small mass-ratio systems, as with the decrease of q, sizes of components become progressively more different leading to shallower eclipses.

Figure 1 contains interesting contribution to the matter of the most common values of the mass-ratio. The broken lines join loci of a_2 and a_4 combinations which can be reached for edge-on orbits (inclination $i = 90°$) for fixed values of the mass ratio, 0.1, 0.3, 0.5 and 1.0. The concentration of observational points close to the origin, with strong fall-off toward the upper right in Figure 1, strongly suggests that large (close to unity) values of the mass-ratio are very infrequent. The mass-ratio distribution is apparently skewed with a preference for small values. We will return to the important matter of the mass-ratio distribution in Sections 5 and 6.

3.   POOR THERMAL CONTACT AND SEMI-DETACHED SYSTEMS

Only the two even cosine coefficients, a_2 and a_4 of the Fourier decomposition of the light curves have been used so far. Now we will consider the first odd term, a_1. Figure 2 shows the two first cosine coefficients, a_1 and a_2, for the full sample of 933 eclipsing systems in the Catalog, divided into groups according to the original OGLE classification. This figure corresponds to Figure 4 in R97 which gave the $a_2 - a_4$ dependence for the whole OGLE sample. The filled circles are EW-type systems according to the OGLE classification, i.e. the contact binaries. Open circles mark EB systems. In the OGLE Catalog, all EB systems have periods longer than one day[2]. Crosses mark all remaining systems with classes E, EA and E?. These systems frequently have eclipses of unequal depths, so that values of a_1 for them may differ substantially from zero. Indeed, as in Figure 4 in R97, we see a clean separation between the contact systems, occupying a band within $-0.02 < a_1 < 0$ (or difference in the eclipse depths less than 0.04 mag), and other binaries which sometimes show strongly negative values of a_1. For contact binaries with good energy exchange between components, the a_1 term is expected to be very small (R93), reflecting almost identical depths of eclipses, in accordance with almost constant effective temperature over the whole contact configuration. The equality of the effective temperatures is a defining feature of the contact binaries of the W UMa-type; it was initially one of the most difficult properties to

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[2]Many of the EB systems classified by OGLE would formally fulfill our Fourier filter for inclusion in the R-sample, but we considered only systems with $P < 1$ day.

explain and led to development of the successful contact model by Lucy(1968).

A closer look at Figure 2 reveals that a certain fraction of systems classified by OGLE as EW or contact binaries also appear to have negative values of a_1. These are the systems of interest in this section. There are two reasons why systems appearing as contact binaries may show differences in eclipse depths: Some of them may be very close, semi-detached binaries (SD), and some may be in contact, but with the energy exchange constricted or diminished for some reason. We will call the latter poor-thermal-contact (PTC) systems. There exists a third reason why systems in good geometrical contact may show deviations from the Lucy model. This is the so-called "W-type syndrome" related to slightly higher surface brightnesses of the less-massive components. It seems to be limited to cool systems and may be related to their chromospheric activity. There have been several attempts to explain this light-curve and a small temperature (about 5%) excess of the less-massive component seems to be preferred (for an exhaustive discussion of the effect for the prototype case of W UMa and for the literature, see Linnell 1991a, 1991b, 1991c). The W-type syndrome is too small to be addressed in this paper and actually must be hiding in the spread of points within $-0.02 < a_1 < 0$ in Figure 2. We are interested here in much larger effects produced by the SD or PTC causes.

It is very difficult to distinguish observationally between contact systems with inhibited energy exchange and semi-detached systems with components close to the inner critical Roche lobe. Systems which show large differences in depths of eclipses, yet appear to be in contact, are sometimes called the B-type systems for their β Lyrae-type light curves. A relation between short-period semi-detached systems and contact systems is expected to exist on the basis of the theoretically predicted Thermal Relaxation Oscillations (Lucy 1976; Flannery 1976; for the most recent review, see Eggleton 1996). In the simplest version, the TRO cycles should consist of two main phases, the good geometrical and thermal contact state and the semi-detached state. In reality, switching between these two may involve the PTC state. Relative durations of the contact and semi-detached branches of the TRO cycles are expected to scale as some larger-than-one power of the mass-ratio: with the contact stage lasting long and the semi-detached stage quite brief. But nothing is known how long the PTC state could last. Observations do not give us a clear picture, mostly because of the poor statistics, spotty coverage of stellar parameter space and difficulties with separation of the SD and PTC cases.

After the work of Lucy & Wilson (1979), which explicitly addressed the question of systems in the "broken-contact" phase, several short-period eclipsing systems with unequal depths of the minima have been studied by Kałużny and Hilditch with their respective collaborators (Kałużny 1983, 1986a, 1986b, 1986c; Hilditch et al. 1984, 1988; Bell et al. 1990). These and other results have been discussed in terms of the temperature difference between the components in the compilation of Lipari & Sistero (1988). These investigations did not give an answer about the evolutionary state of such systems. Some of them seem to be genuine PTC contact systems with components of unequal temperatures, some may be in the semi-detached state (with either the more- or less-massive components filling their critical lobes), mimicking contact systems. Usually,

a full set of photometric and spectroscopic data is needed in individual cases. One property however is clear: Irrespectively what produces the large differences in depths of eclipses, such systems are very rare, even in spite of certain advantage in discovery relative to normal contact systems. We see them only among systems having orbital period longer than a certain threshold value: the SD or PTC binaries do not occur among very cool, short-period systems. The border line is currently at about 0.4 days, defined by the shortest-period PTC system known at this time, W Crv with $P = 0.388$ day (Odell 1996).

The OGLE data give us a first chance to look into statistics of the occurrence of unequally deep eclipses. However, we should note an important limitation of results based solely on such photometric data: Unless we see total eclipses, we do not know which star, more or less massive, is eclipsed at each minimum. In view of the surface-brightness deviations from the contact model, this is a serious complication. Here, we follow the conventional way of counting phases, from the deeper minimum, i.e. from the eclipse of the hotter star. A more meaningful convention, normally used in light-curve synthesis programs, would be with phases counted from the eclipse of the more-massive star.

Figure 3 shows the same data as in Figure 2, but for the R-sample. Its sub-sample with the distances smaller than 3 kpc (the reddening model with $d_0 = 2$ kpc, designated as R_2, see the Introduction), will be from now on called the "local sample". As was discussed in R97, this distance delineates a volume-limited sample of contact systems in the OGLE data. We can immediately see that contact systems with large differences in depths of minima are all distant, hence intrinsically bright. Among the 98 R-sample systems to 3 kpc, only 2 have $a_1 < -0.03$ (difference in depth larger than 0.065 mag), so that the phenomenon of unequal eclipse depths is very rare and affects only some 2% of the systems. For the full R-sample extending all the way to the Bulge, with the same magnitude-difference threshold, we have 36 among 388 or 9% of all systems. The two systems in the local R-sample with unequal depths of minima are #3.012 and #6.005. Their orbital periods are 0.370 and 0.698 days. The first system sets a new short-period limit of the occurrence of the phenomenon, replacing W Crv with 0.388 day. The light curve for #3.012 is shown in Figure 4. The system #6.005 (discussed in the next section, its light curve is in Figure 11) is the only nearby systems with large eclipse-depth difference and relatively long orbital period.

The results on a_1 for the R_2 sample are shown as an orbital-period dependence in Figure 5 and as the intrinsic-color dependence in Figure 6. As was discussed in R97, the range of the observed intrinsic colors for most systems is relatively narrow, due to their concentration in the Turn-Off Point region for an old stellar population. The distribution is additionally compressed on the red side due to progressive elimination with distance of faint, red systems from the full, magnitude-limited R-sample. As we already said, the local systems, with a relatively wider range of colors, show small values of a_1. This applies also to the reddest binaries which we will discuss now in a small detour from the main subject.

The three reddest systems in Figure 6, with the observed colors $V - I > 2$ and the intrinsic colors $(V - I)_0 > 1.5$ belong to the local sample with distances of 600 – 1400 parsecs. These small distances may be erroneous if the colors are red due to some observational problems rather than to genuinely low effective temperatures, as very red colors lead to low intrinsic luminosities in our absolute-magnitude calibration. The red colors are really unusual when compared with the current short-period, red-color limit for the contact binaries determined by CC Com with $P = 0.2207$ day, $B - V = 1.24$, $V - I = 1.39$ and the spectral type about $K5$ (Rucinski 1976; Bradstreet 1985). All three systems, #3.053, #7.112 and #8.072, are quite unremarkable as far as their light curves are concerned. Only #8.072 has a short orbital period of 0.284, in some accord with the color, but even in this case the color is well beyond what has been observed before for contact systems: $V - I = 2.04$ and $(V - I)_0 = 1.77$, implying $(B - V)_0 \simeq 1.4$. #3.053 is unusual in that it has a moderately long period of 0.466 day in combination with a very red color: $V - I = 2.59$ and $(V - I)_0 = 2.31$. The corresponding $(B - V)_0 \simeq 1.5$ implies the spectral type as late as M1 or M2. The light curves of #3.053 and #8.072 have moderately large amplitudes of $\Delta I = 0.45$ and 0.54 so that blending of images with other, very red stars would be difficult to postulate. Such blending may be the explanation for the low amplitude of #7.112 ($\Delta I = 0.17$), which is unusually red ($V - I = 2.58$) for its orbital period of 0.590 day. All three systems require further observations. We note that the possible error of 0.1 in the OGLE mean colors for red stars cannot be an explanation here, as the stars are simply too red.

4. LIGHT-CURVE ASYMMETRIES

Light-curve asymmetries are fairly common in contact binaries. The difference in heights of minima is sometimes called the O'Connell effect (for earlier references, see Linnell 1982 and Milone et al. 1987). There is no generally accepted interpretation of such asymmetries, but the most obvious explanations would be in terms of stellar spots or some streaming motions deflected by Coriolis forces. The starspot explanation seems to work well in cool systems which are expected to be very active. Little is known about causes of persistent asymmetries for systems of spectral types earlier than F-type since large magnetic spots would be difficult to imagine to exist on these stars. Sometimes, the asymmetry is so large that it must be caused by some streaming/accretion phenomena. Of particular importance here is the short-period ($P = 0.301$ day), late-type (K3V) detached system V361 Lyr which shows a huge asymmetry (Kałużny 1990, 1991; Gray et al. 1995) apparently due to an accretion process currently taking place between components[3]. The maximum after the deeper eclipse is higher in this case, indicating in-fall on the cooler component as the most likely direction of the gas streaming.

No statistics are currently available as to how large are the asymmetries and how often do they

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[3]To the author's knowledge, no radial-velocity of this important system has been performed so that all inferences about it are still based solely on photometric data.

occur. Selection of sky-field objects for individual observations is obviously highly biased. A large sample, such as the OGLE sample, which can be subdivided into magnitude- and volume-limited ones, is of great usefulness here. Before presenting the results, the same warning as in Section 3: Our photometric sample suffers from the ambiguity in the origin of phases by half of the orbital period. If the asymmetry phenomena are driven by the Coriolis force, they are expected to show signatures possibly related to the relative masses of components eclipsed at each minimum. We do not have this information, but instead we have information about the relative effective temperatures of components. These might correlate with the masses, but this is not obvious that this must necessarily be the case.

The asymmetries have been analyzed in the simplest possible way by inclusion of one sine term in the Fourier decomposition. No major differences exist in the results for the O- and R-samples, so that only the latter sample will be discussed here. Figure 7 contains the sine coefficients, b_1, plotted versus the largest cosine coefficient a_2. The sign of b_1 indicates which maximum is higher after the deeper eclipse. No obvious correlation between both coefficients seems to be present. However, the next Figure 8, with the dependence of b_1 on the orbital period, brings up an interesting property: While for the local, mostly short-period systems we see about equal numbers of positive and negative values of b_1, for systems with periods longer than about 0.4 day, the positive asymmetries dominate. This effect is not very strong, but is definitely present, as is shown in the histogram of the b_1 coefficients for 302 systems of the R-sample with $P > 0.4$ day in Figure 9. The distribution is only mildly asymmetric: the mean is slightly positive, $+0.0013$, and the skewness is not significant (0.33 ± 0.22). However, if we treat the both branches as independent distributions, the χ^2 test gives probability of only 0.0047 that the differences in numbers in 8 bins of $|b_1|$ could be due to random fluctuations. If we eliminate the two bins close to the origin (-0.002 to $+0.002$) which are most susceptible to random fluctuations, then the probability that the two branches are different due to a statistical fluctuation decreases to as little as 0.0026. Thus, the difference in the two sides of the distribution is significant at the level of 0.995 to 0.997. We see no way how the asymmetry in the distribution of b_1 could be generated by ways of data reductions. It must be real. Apparently, for long-period systems, incidence of the elevated maximum after the deeper eclipse is higher than the opposite case.

The most obvious candidate for the cause of the asymmetry in the distribution of b_1 would be the Coriolis force acting on gas streams between components. The sense of the deviations induced by the Coriolis force is such that – if we see hot spots due to accretion phenomena (as seems to be the case for V361 Lyr) – the visibility of spots *before the secondary minima* implies that the spots are located on the cooler components. This would be exactly what should happen for binaries in the semi-detached state with more massive, hotter components losing mass for their less massive companions. At this point, without spectroscopic data, this is a hypothesis rather than a definite statement.

If some of the light-curve asymmetries are caused by mass-transfer phenomena in semi-detached systems of similar type, then we could expect a correlation between the sense and size

of the asymmetry and the difference in the depths of eclipses. Such a correlation cannot be very tight as it would reflect the percentage admixture of the semi-detached systems among all systems showing the light curve asymmetries. The scatter plot for $a - 1$ and b_1 is shown in Figure 10 where some correlation between these quantities is indeed visible. The formal linear (Pearson) correlation coefficient -0.29 ± 0.08 indicates presence of a correlation with a relatively high significance (probability that the observed value resulted from a correlation of independent distributions is at the level 0.001). The three systems with the largest deviations in Figure 10 ($a_1 \leq -0.05$ and $b_1 \geq 0.01$), and thus the best candidates to be the semi-detached systems with mass-transfer are the following: #0.111, #3.094 and #8.133. Their periods are relatively long, 0.639, 0.912 and 0.509, which coupled with their blue colors gives the distances 4.5, 7.9 and 5.3 kpc (under the R_2 assumption). System #3.094 was suggested in R97 to be in the Galactic Bulge.

A typical light curve for a system showing a large asymmetry is shown in Figure 11. T he system is #6.005 with the orbital period of 0.698 day which was already mentioned in the previous section as one of the two nearby systems with a large difference in the depth of the eclipses. Thus, this system directly shows the correlation between a_1 and b_1 and may be taken as a prototype of the class.

The dependence of the b_1 coefficient on the intrinsic color is shown in Figure 12. This plot seems to indicate that for very cool systems, negative b_1 may be more common, but because of the small number of such systems, the significance to this indication is low.

5. AMPLITUDE DISTRIBUTION

Amplitudes of light variations of contact binaries contain an important information on the mass-ratio (q) distribution: Large amplitudes can be observed only for large mass-ratios, close to unity; for small mass-ratios, only small amplitudes can be observed. This simple relation is illustrated in Figure 13 for a few theoretical distributions calculated on the basis of R93 for mass-ratios within narrow ranges of $\Delta q = 0.1$ (every second interval in q). We see not only the obvious dependence between the largest possible amplitudes and the mean value of q, but also the effects of total eclipses producing similar amplitudes for large fractions of systems within q-dependent ranges of inclinations close to $i = 90°$. The distributions in Figure 13 cannot be obviously observed as weighting by the mass-ratio distribution $Q(q)$ occurs. Exactly this weighting opens up a possibility of observational determination of $Q(q)$ which would be of great importance for our understanding of formation and evolution of contact binary stars.

Application of the line of reasoning described above is not easy, even for such a large sample as the one being analyzed now. First of all, for an inverse problem of determination of $Q(q)$ from the amplitude distribution, $A(a)$, very good statistics for *each bin* of A is needed. This is the main obstacle why our attempts at determination of $Q(q)$ through a solution of the integral equation $A(a) = \int Q(q)K(q, a)\, da$ have been so far unsuccessful and will not be discussed here. The second

problem is that the magnitude-limited sample is biased by the presence of distant, bright systems. As was shown in Section 3, these are systems with the highest frequency of occurrence of unequally deep eclipses. For such systems, primary minima are deeper and secondary minima are shallower than for good thermal contact models leading to a corrupted statistics of the amplitudes (although use of the a_2 coefficient could possibly help here). If we eliminate those bright and distant systems, and utilize only the systems of the local sample (which seems to give a fair representation of most typical systems), the sample becomes too small for definite applications. Finally, the distribution of the third geometrical parameter, the degree-of-contact, is not known but it does have some influence on the amplitude distribution, judging by the theoretical results, as in Figure 14.

Figure 14 gives the observed distributions $A(a)$, where $a = \Delta I$ in the OGLE data, as well as some additional theoretical predictions. The latter are shown by lines for the flat mass-ratio distribution $Q(q)$ and for two cases of the degree-of-contact, $f = 0$ and $f = 0.5$ (upper panel), as well as for for a strongly falling distribution, $Q(q) = 1 - q$ with $f = 0$ (lower panel). These theoretical distributions show some sensitivity to the assumptions on the shape of $Q(q)$ and on the value of f, but look very different from the observed distributions. The full R-sample (upper panel) and the local R_2 sample to 3 kpc (lower panel) are shown as histograms. They indicate dominance of low-amplitude systems. Large amplitude systems are almost non-existent in the OGLE sample, which agrees very well with the old open-cluster data (Kałużny & Rucinski 1993; Rucinski & Kałużny 1994). The most obvious explanation for dominance of small amplitudes would be by a $Q(q)$ distribution which steeply increases for small values of q.

The OGLE sample gives us a very different picture than for the whole sky. In the sky field, we observe several bright contact systems which have large variability amplitudes, and some of them indeed have spectroscopically-determined mass-ratios very close to unity. The extreme cases are SW Lac with $q = 0.73 \pm 0.01$ (Hrivnak 1992), OO Aql with $q = 0.84 \pm 0.02$ (Hrivnak 1989) and VZ Psc with $q = 0.92 \pm 0.03$ (Hrivnak & Milone 1989). Thus, although $q = 1$ apparently never happens, some mass-ratios can be quite close to unity. The large values of q for the field systems are consistent with the amplitude distribution for the whole sky which peaks at $a \simeq 0.55$ and extends to amplitudes as large as one magnitude (Rucinski & Kałużny 1994; shown as a dotted histogram in the upper panel of Figure 14). Judging by the OGLE data, these are atypical cases, emphasized by the ease with which large-amplitude systems are detected in the sky surveys. More typical contact binaries show small variation amplitudes which are related to their small mass-ratios. Such systems remain to be discovered among bright stars of the sky field.

6.   MASS-RATIO DISTRIBUTION FROM TOTALLY ECLIPSING SYSTEM

Since inversion of the amplitude distribution for derivation of $Q(q)$ presents several problems, one can consider a different approach: Mochnacki & Doughty (1972a, 1972b) described a method of element determinations utilizing the angles of internal eclipse contact for totally eclipsing systems. This method can obviously be applied only to a sub-sample of all systems, but has

an advantage of being very weakly sensitive to the degree-of-contact: Basically, for contact configurations, only two geometrical parameters, q and i, determine the the angle of the inner eclipse contact, $\phi = \phi(q, i)$.

The method of determination of the mass-ratio distribution $Q(q)$ would be quite simple: The distributions $Q(q)$ and of the orbital inclinations $I(i) = \sin i$ must be statistically independent. Thus, from the distribution of the inner-contact angles, $\Phi(\phi)$, one could determine $Q(q)$ by solving the integral equation:

$$\Phi(\phi) = \int_{q(0^\circ,\phi)}^{q(90^\circ,\phi)} Q(q) \, \sin[i(\phi,q)] \, |\partial i(\phi,q)/\partial\phi| \, dq$$

At present, we have not been able to apply this approach because the sample of totally eclipsing systems among the OGLE systems was too small. Also, because of the relatively large observational errors of about 0.02 mag, it was difficult to set up an automatic-selection process of finding totally-eclipsing systems. However, the approach holds a great potential and, in the future, should be used on large samples of well observed systems. Since, as we have shown in Section 5, the mass-ratio distribution must be skewed to small values of q, a relatively large fraction of contact binaries should show total eclipses.

7. SUMMARY

The paper contains analysis of light curves of the W UMa-type binaries in the OGLE Catalog of Periodic Variable Stars for fields BWC – BW9. It is a continuation of R97, but concentrates on properties of the contact systems, rather than on their usefulness for galactic-structure studies. The important result of R97 that the contact binaries belong mostly to the Turn-Off Point population of old stars is not amplified here; the stress is on structural properties of the systems, as they can be gauged using simple methods of light curve characterization. Since the observations have moderate accuracy of about 0.02 mag, the low-order Fourier decomposition was judged to provide an adequate tool for such a characterization.

Rough estimates of the degree-of-contact, obtained using the a_2 and a_4 cosine coefficients confirm that the most frequent values are concentrated with $0 < f < 0.5$, suggesting weak contact. The quality of this determination is low and it is somewhat qualitative. A much more interesting results came from the analysis of the differences in depths of eclipses which was based on the first cosine coefficient, a_1. In the volume-limited "local" sample to 3 kpc, among 98 systems, only 2 show appreciable depth differences, so that good thermal and geometrical contact is a norm rather than an exception. One of these systems, #3.012, sets a new short-period limit of 0.370 day for the occurrence of unequal eclipses. For the full magnitude-limited sample, which is favorably biased toward the intrinsically bright, distant systems, incidence of the large depth differences is more common with some 9% showing differences in depths of minima larger than 0.065 mag ($a_1 < -0.03$). Some of these may be in good geometrical contact and with the inhibited energy

exchange, the poor-thermal-contact systems (PTC), but some may be actually semi-detached (SD) systems, very close to the contact configuration. The present data, in only one spectral band and without spectroscopic support, do not permit to distinguish between these possibilities in individual cases.

Analysis of the light curve maximum-light asymmetries – measured by the first sine term b_1 – which weakly (but significantly) correlate with the differences in the depths of the eclipses for periods longer than about 0.4 day, suggests that the semi-detached state with mass-exchange is common among systems with unequally deep eclipses. The asymmetries would be then caused by mass-exchange streams impinging outer layers of cooler components. Since the asymmetries may be also caused by photospheric spots, the statistics of asymmetries cannot be used to determine the number of SD systems mimicking contact binaries. The second system in the local sample which shows a large difference of the eclipse depths, #6.005, is a perfect illustration of the correlation between the difference in the depth of eclipses and the sense of the light curve asymmetry. The overall rarity of systems with large eclipse-depth differences suggests that the admixture of semi-detached or poor-thermal-contact systems to the totality of contact systems is very small. These are however the intrinsically brightest systems which are visible to large distances so that they are preferentially represented in magnitude-limited samples. In the future, it would be highly desirable to obtain light curves in a few spectral bands, say in the U, B, V and I filters. This would permit decoupling of the effects of the accretion streams and temperature differences between components from the geometric issues of contact versus semi-detached configuration.

Presence of only two SD/PTC systems among 98 contact binaries of the local sample can explain why we do not see such systems among bright stars. Using the apparent frequency of contact binaries found in R97 of $1/250 - 1/300$, we can estimate the expected frequency of SD/PTC systems among Main Sequence dwarfs to be about $1/12{,}500 - 1/15{,}000$; since this estimate is based on 2 cases, it carries a Poissonian uncertainty of a factor of about 1.4. A complete sample of bright stars to $V \simeq 7 - 8$ is accessible from the Hipparcos Input Catalogue (CD-ROM Version, Turon et al. 1994). By counting luminosity-class stars IV and V in the HIP, within the color range where contact binaries occur (R97), $0.4 < B - V < 1.2$, to the successively deeper limiting magnitudes $V = 5$, 6, 7, and 8, one obtains 128, 497, 1620 and 4561 stars. Thus, no SD/PTC systems are expected to the brightness levels at which the Hipparcos sample starts showing selection effects. However, by going one magnitude deeper, we could expect some 3–4 time more stars, so that we should be able to detect one or two SD/PTC system. This is well confirmed by the actual numbers as the brightest among such systems, AG Vir and FT Lup, appear at $V = 8.5$ and 9.2 (Kałużny 1986b, 1986c). As an aside, we mention here that the sample of bright contact binaries, when compared with the Hipparcos numbers, fully confirms the apparent frequency of $1/250 - 1/300$: To the same V-magnitude limits as above, the variable-star lists contain 3, 3, 6, and 14 contact systems.

The distribution of the variability amplitudes suggests that the mass-ratio distribution $Q(q)$ increases toward small values of q. Since the increase appears for $\Delta I < 0.6$, but is modified by

discovery selection already below $\Delta I < 0.3$, the present distribution is just too narrow to attempt a determination of the mass-ratio distribution $Q(q)$. A distribution emphasizing low values of q should lead to frequent occurrence of totally eclipsing systems. Such systems may allow an entirely independent determination of $Q(q)$ based on the angles of the inner eclipse contacts. We note that the sky-field is surprisingly devoid of low-amplitude contact systems; we suspect that they have been simply overlooked in non-systematic surveys, a result which has led to an over-representation of large-amplitude contact systems.

Note: The data on the W UMa-type stars in the OGLE survey of fields BWC to BW8 which were used in R97 and in this paper are available from the author or from http://www.astro.utoronto/~rucinski/rucinski.html

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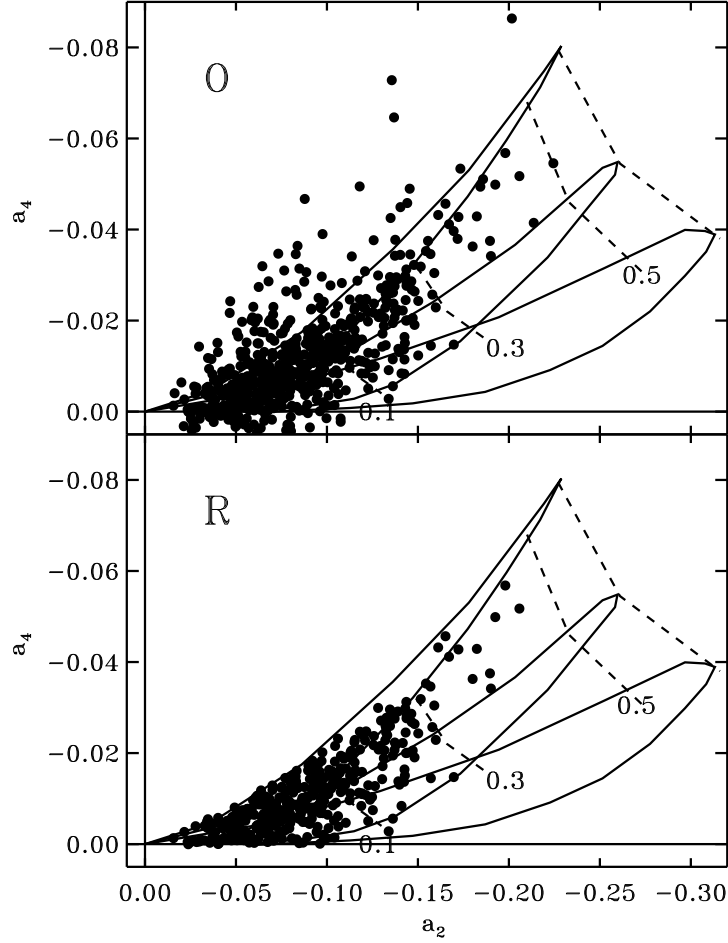
Fig. 1.— Relations between the even Fourier coefficients a_2 and a_4 for the O- and R-samples of contact binaries. The three elongated regions give envelopes of all combinations of orbital inclinations and mass-ratios considered in R93 for three values of the degree-of-contact parameter f. The upper, thin one is for the inner (marginal) contact ($f = 0$) and the lowest, broad one is for the outer contact ($f = 1$); the intermediate is for $f = 1/2$. The broken lines give schematic locations of the combinations of the coefficients for $i = 90°$ for four values of the mass-ratio q of 0.1, 0.3, 0.5 and 1.0.
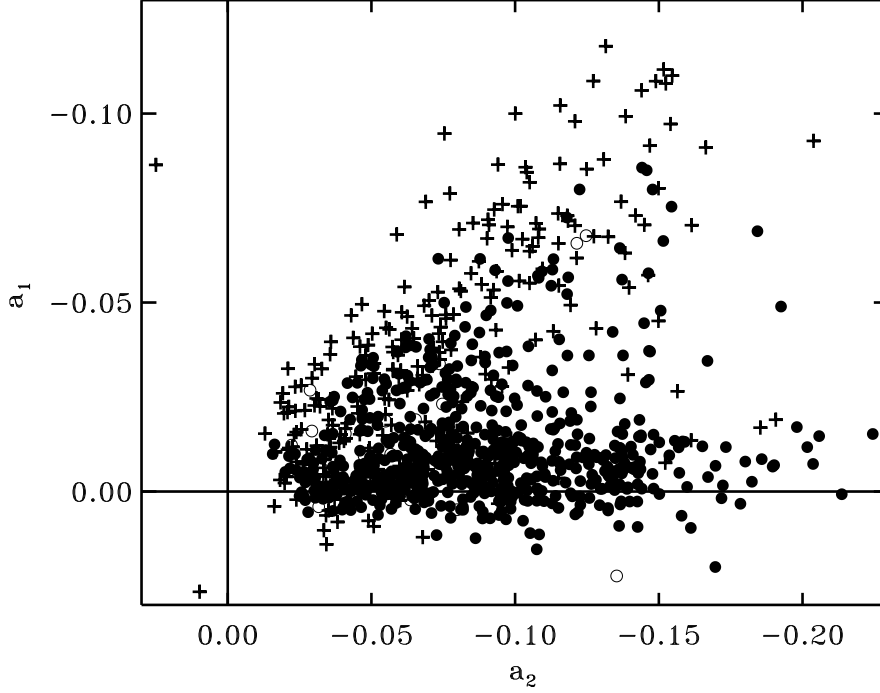
Fig. 2.— Relations between the Fourier coefficients a_1 and a_2 for all 933 eclipsing systems as classified in the OGLE Catalog (fields BWC – BW9, the same as in R97). a_1 is sensitive to the difference in depth of eclipses while a_2 is proportional to the overall amplitude of light variations. The symbols are as follows: filled circles – W UMa-type systems (EW), open circles – β Lyrae-type systems (EB, in the OGLE classification, all these systems have periods longer than one day), crosses – all remaining eclipsing systems (E, EA, E?). Note that most contact systems show small eclipse-depth differences within $-0.02 < a_1 < 0$. The group of systems showing larger differences may consist of genuine contact systems with poor thermal contact (PTC) and of short-period semi-detached (SD) systems mimicking contact systems.
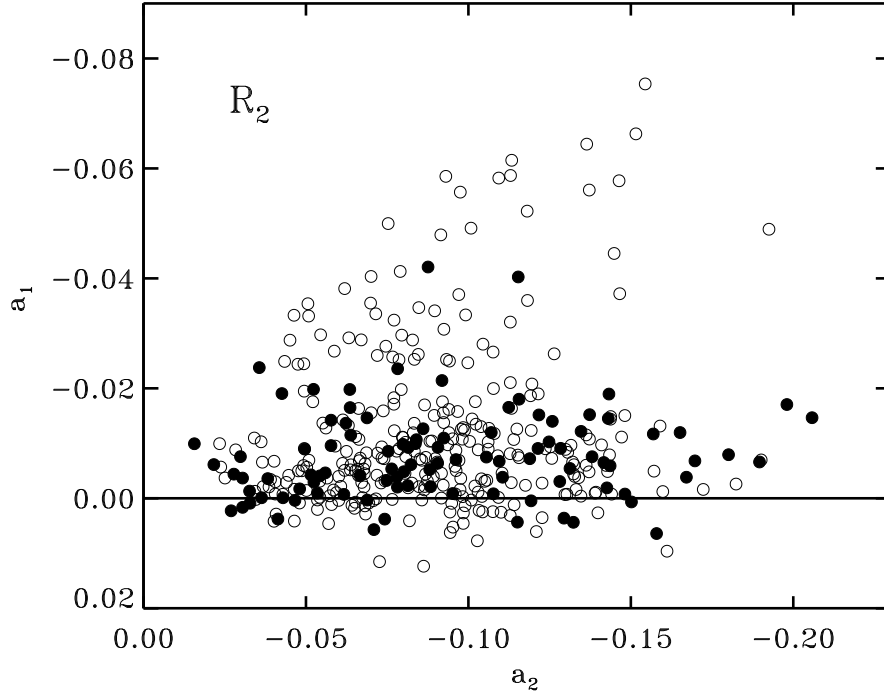
Fig. 3.— The same plot as in Figure 2, but limited to the contact systems of the R-sample. In this and in subsequent figures, systems forming the "local sample" to 3 kpc, which we think is complete, are marked by filled symbols. Note that PTC and SD systems are rare in space: In the local sample consisting of 98 systems, there are only two cases with $a_1 < -0.03$ or, equivalently, with depths of eclipses differing by more than 0.065 mag.
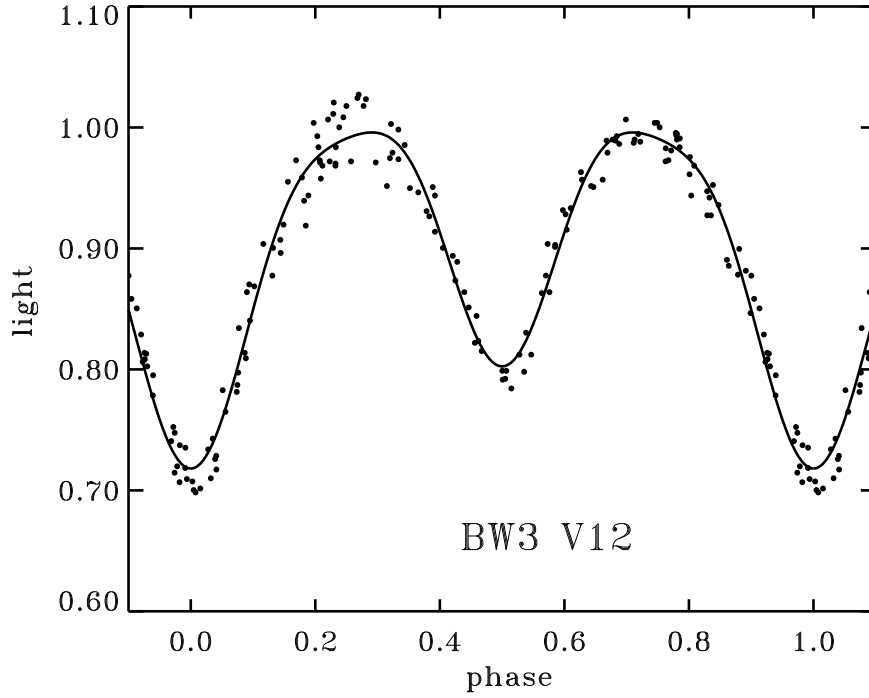
Fig. 4.— The light curve for #3.012 which is presently the shortest-period (0.370 day) known contact binary showing a moderately large eclipse-depth difference. The line gives the fit based on the first five cosine terms (zero to four). Note the scatter of points in the first maximum which might indicate a transient asymmetry, similar to those frequently observed among systems showing differences in depths of eclipses (see the next section).
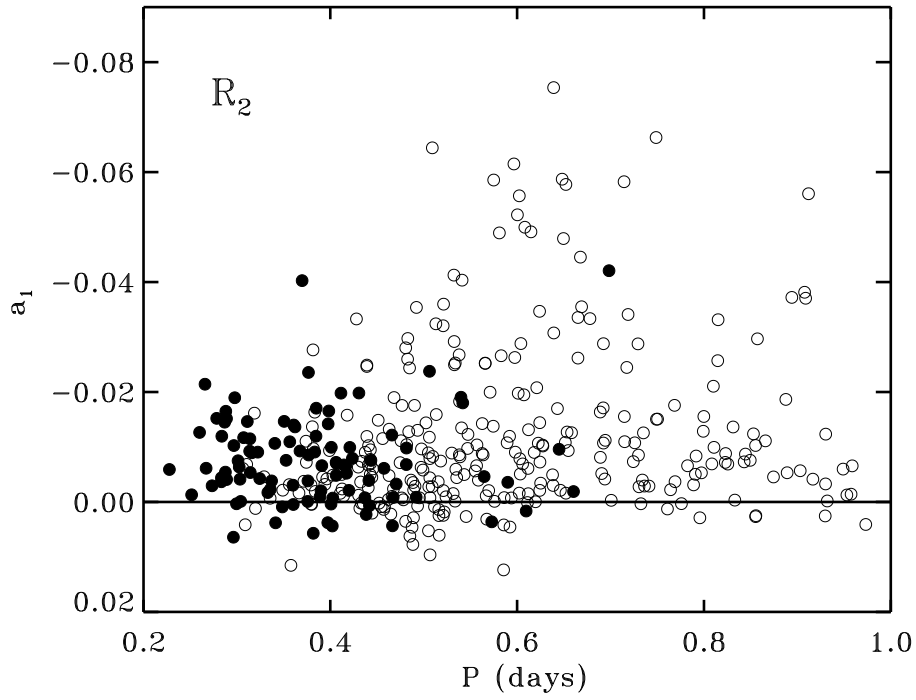
Fig. 5.— The a_1 coefficients plotted versus the orbital period in the same format as in Figure 3. Note the increase in frequency of occurrence of unequally-deep minima among systems with longer periods. The two local systems are #3.012 at 0.370 day and #6.005 at 0.698 day.
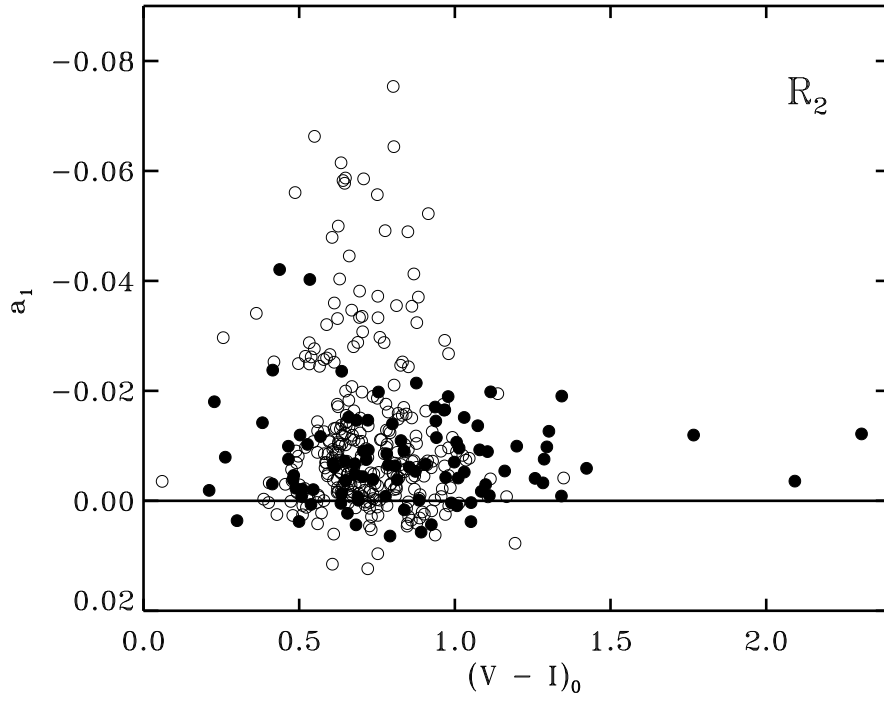
Fig. 6.— The same data as in Figure 5 plotted versus the intrinsic color. The three very red systems are described in the text.
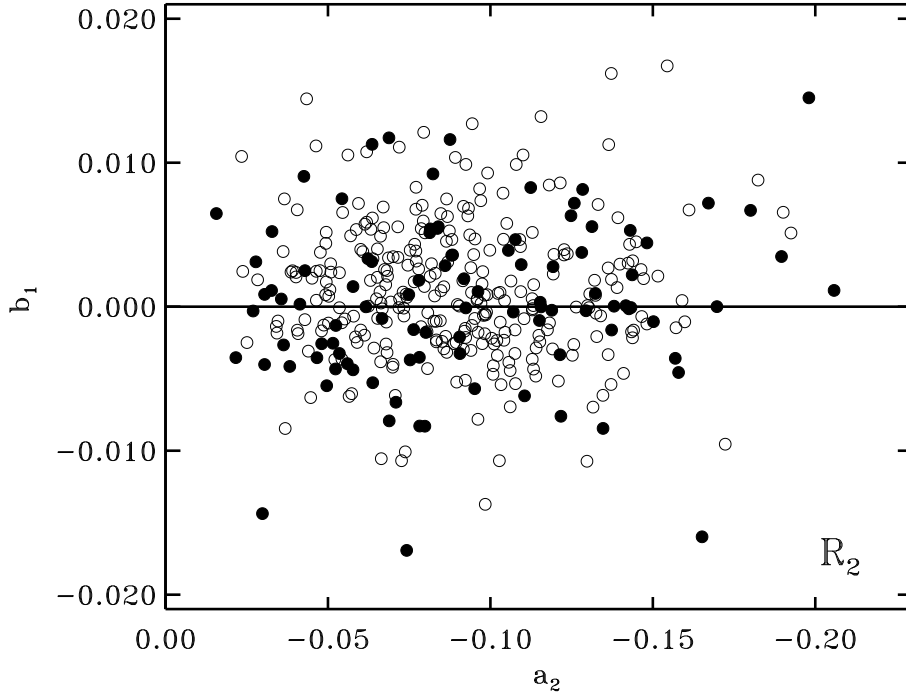
Fig. 7.— The first sine coefficient in the Fourier representation of the light curves, b_1, is shown here versus the largest even coefficient a_2. Although the diagram seems to indicate a random scatter in b_1 without any sign preference, the positive values of b_1 are more frequent than the negative values for intrinsically-bright and more distant systems with long orbital periods; see the next figure.
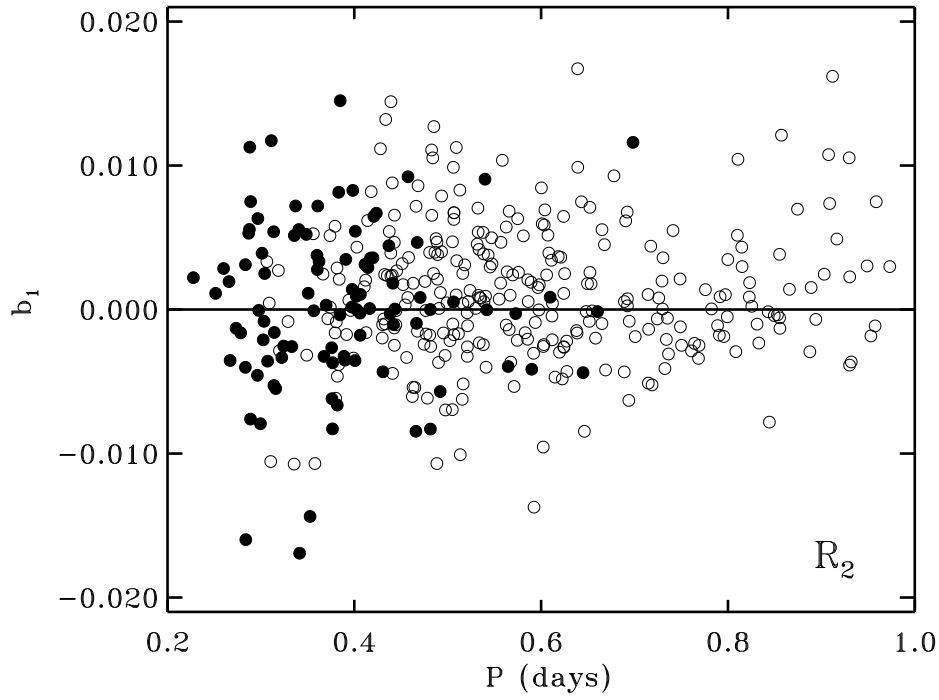
Fig. 8.— Positive values of the sine coefficients b_1 are systematically more common among contact systems with orbital periods longer than about 0.4 days. The local-sample systems, which in their majority have periods shorter than 0.4 day, seem to show a symmetric distribution of the b_1 values.
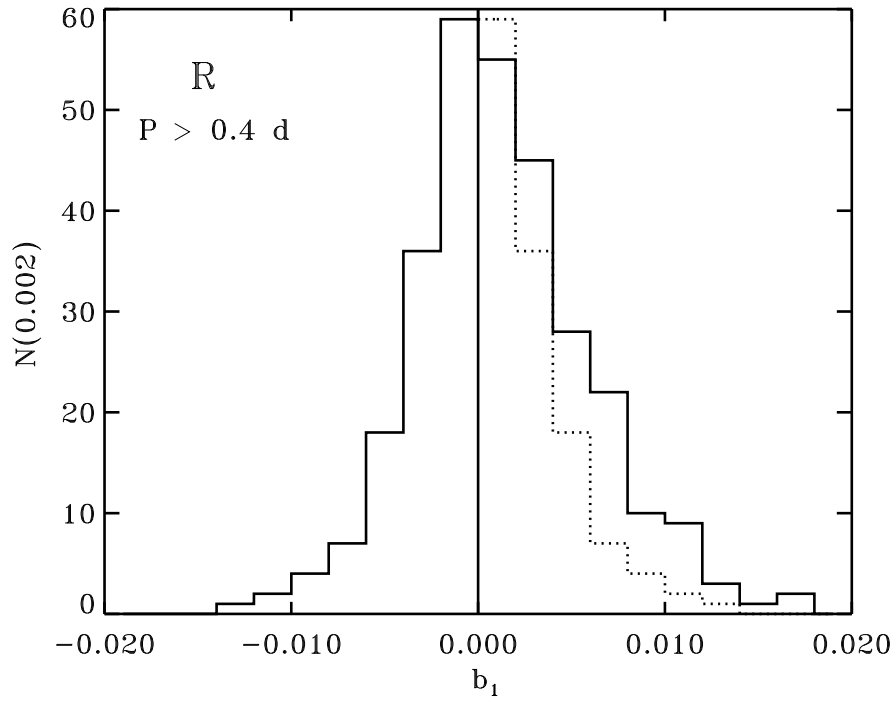
Fig. 9.— The histogram of b_1 for all systems in the R-sample with the orbital periods longer than 0.4 day. The dotted line visualizes the difference of the two sides of the distribution by giving the reflection of the negative part of the histogram into the positive side.
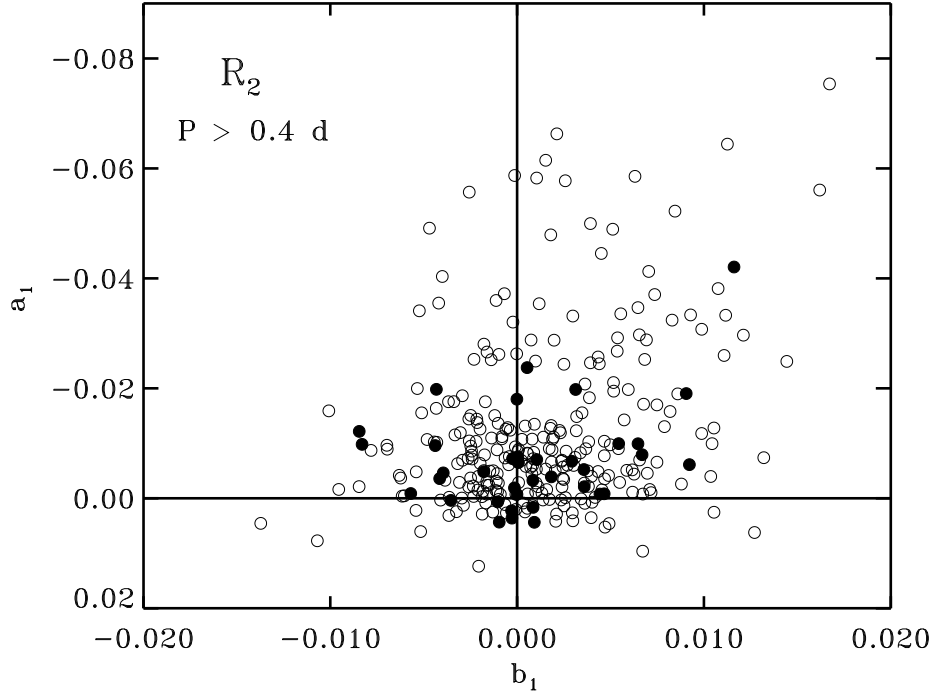
Fig. 10.— Relation between b_1 and a_1 coefficients for the R-sample systems with periods longer than 0.4 day. b_1 measures the light-curve asymmetry and a_1 measures the difference in depth of eclipses. The correlation between these quantities has a 3-sigma level significance. Note the single filled circle in the upper right quadrant which corresponds to the system #6.005; its light curve is shown in the next figure.
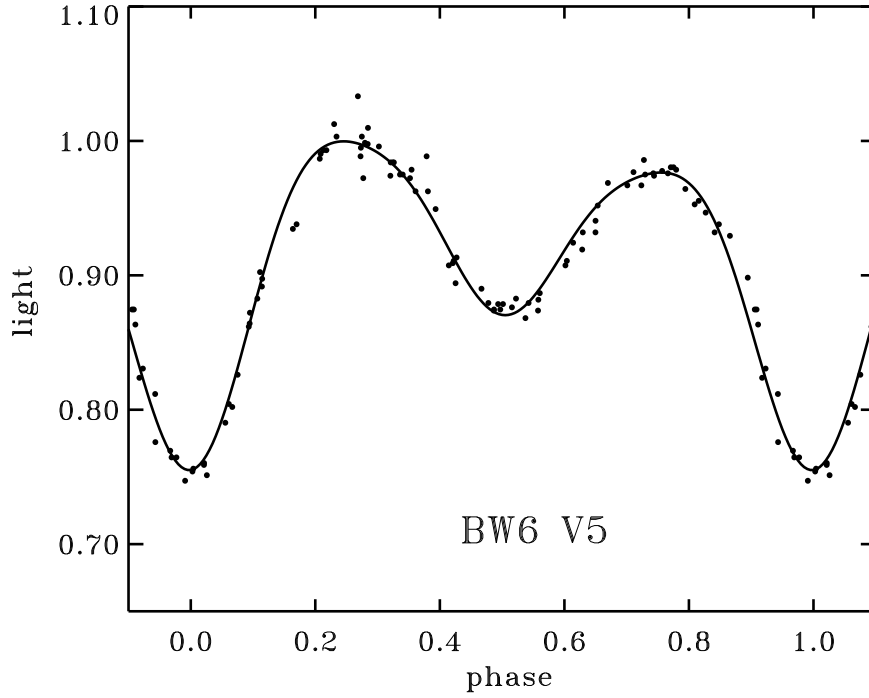
Fig. 11.— The light curve for the binary #6.005 (orbital period 0.698 day) which shows the largest asymmetry of maxima among the systems of the local sample. This system shows also a large difference in the depth of the eclipses. Note the sense of the asymmetry which is apparently the most common among similar systems. The fit, shown by a continuous line, consists of 5 cosine terms (zero to four) and one sine term.
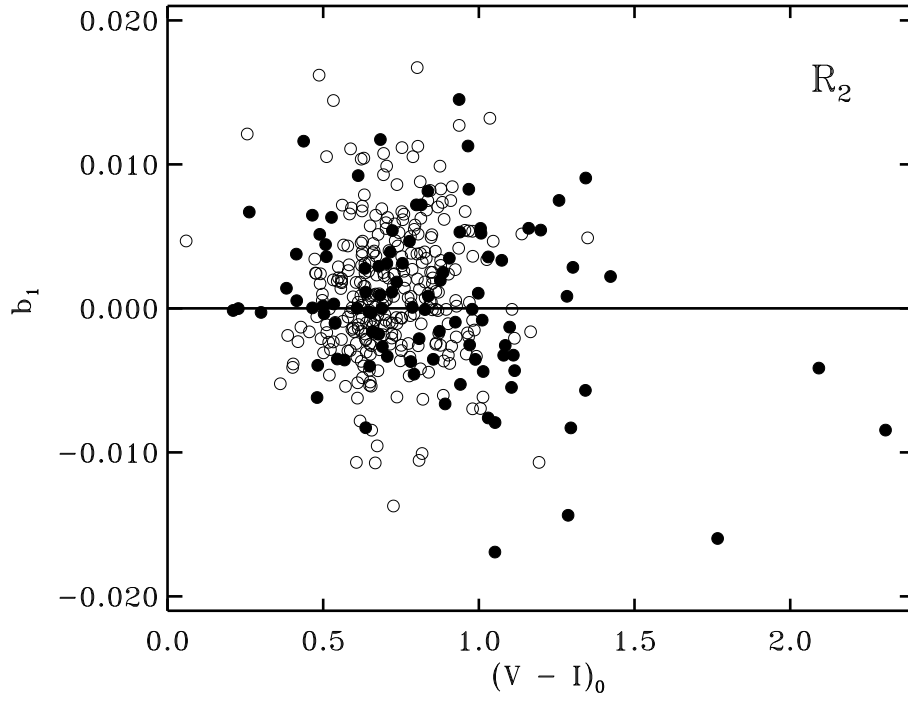
Fig. 12.— Relation between b_1 and the intrinsic colors. Note that all of the few very red systems of the local sample show negative values of b_1.
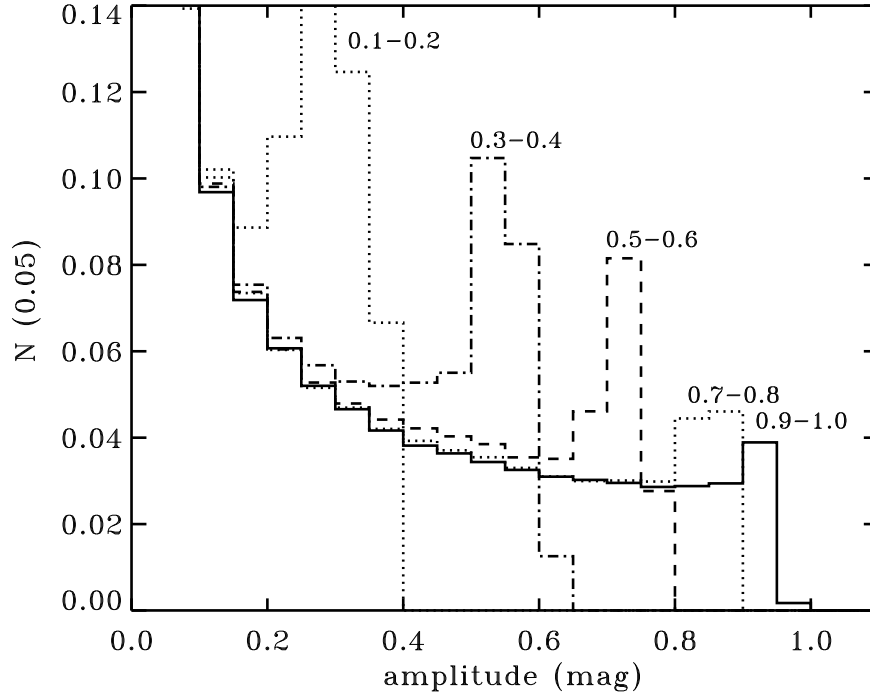
Fig. 13.— The mass-ratio distribution is expected to strongly influence the distribution of the variability amplitudes. Here, the theoretical distributions, based on the data in R93, are shown for idealized $Q(q)$ distributions consisting of flat segments within $\Delta q = 0.1$ ranges of q, as labeled in the figure. For a broad $Q(q)$, the broad spikes due to higher frequency of total eclipses for narrow ranges in q become diluted in the combined amplitude distribution. The low amplitude end of the distribution is practically insensitive to changes in $Q(q)$.
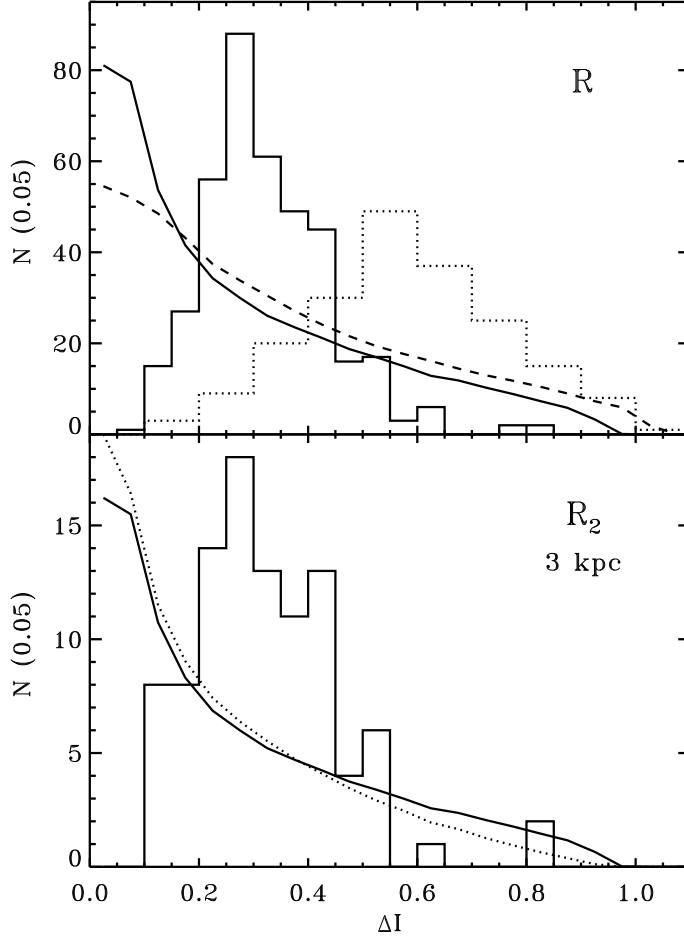
Fig. 14.— The observed amplitude distributions for the R-sample (histograms) are compared here with the general field distribution (dotted histogram) and with some theoretical predictions (lines). The upper panel gives the histogram for the whole R-sample while the lower panel is for the local sub-sample to 3 kpc. The theoretical predictions are shown by lines with an arbitrary normalization to the total number of 100 systems. For both panels, the same continuous line gives the expected amplitude distribution for $Q(q) = const$ and for the inner contact, $f = 0$. Additionally, the broken line in the upper panel gives the case $Q(q) = const$ and $f = 0.5$, whereas the dotted line in the lower panel gives the inner contact case, $f = 0$, but for a falling mass-ratio distribution, $Q(q) = 1 - q$. Note the weak sensitivity of the theoretical distributions to these modifications. The theoretical distributions appear to be very different the observed distributions which steeply rise for amplitudes below $\Delta I < 0.6$; unfortunately, the latter are affected by the detection incompleteness for $\Delta I < 0.3$ (R97). The sample of the sky-field systems (dotted histogram) is heavily biased by the ease of detection of large amplitude systems which do exist but are apparently very rare (see the text).